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# Mind Alone

The relationship that obtains between the mind and the brain is at the very heart of our human nature, a dual of man and animal. A recent experience has brought into sharp focus the workings of this partnership.

Many years ago I subjected myself to the rigors of graduate study in mathematics, and took a doctorate in mathematics from Northwestern University. Even when I started on that course, I was aware that I really had no special desire to be a mathematician. Instead, I had a vague conviction that I wanted to know what it is that mathematicians know. Whether that thought was prescience on my part or was simply blindness I do not know, but it does correlate with the way things actually worked out. Shortly after graduation I found myself out of work and my solution was to return to graduate school in business. Since then, business and economics have been my career, but my fascination with mathematics never left me. Mathematics is pure abstraction, and my chosen field within math is the most abstract of all: geometry. It is the science of the relationships between "things," taking the term "relationship" in its most literal, geographical sense. Geometry replicates an infinite variety of other, immaterial relationships, so we are not in the end limited to lines and circles in the strict sense, but the world of the geometrician – as distinct from that of the scientist who applies his findings – is content not to question the geometrical or "geographical" realization of the subject matter. We actually like lines and circles.

This preamble is moreover not merely a nostalgic exercise. It goes to the heart of the point I wish to make here. It describes me as a true mathematical amateur, a "professional" amateur so to speak. I am professionally excused from having to think about geometrical problems that occupy the best efforts of my professional colleagues, and leaves me the extraordinary freedom to pursue those questions that catch my fancy. The price of this freedom is that I have no expectation of being paid for my work, but that is not a practical problem in my case. I certainly do not oppose the idea of paying professional mathematicians for their work, but note that for me it is a great convenience not to depend on that as my livelihood.[1] My mathematical work is thus purely individual. I have not tried to keep up with the state of the art; the problems I work on are ones that only I have an interest in, at least until I come to the point of trying to interest mathematical journals in publishing them. Everything I think about – every piece of "stuff" that I turn over and investigate in my mind – is entirely of my own creation. It is that attribute of the work that has brought what follows to light.

## An interesting problem explained

It is not my purpose in this note to teach higher math. The reader will have to undertake to gain that skill on his own without my help. Nonetheless, it is necessary to introduce a bit of math because it helps to explain what is my point here.

As a result of other work in geometry I formulated the following interesting – at least interesting to me – question. Let us agree that the term "coloring" a geometrical sphere means to assign the points on the sphere to distinct classes – the "colors" – with the property that every point goes in one and only one of them. Every point gets a color. There are no other rules. Which color is assigned to point A does not in any way limit the assignment of a color to any other point B. Now, we want to focus on cases in which there are only a finite number of different colors, and for simplicity in what follows I will use the whole numbers to name them. So we are talking about color #1, color #2, and so on. Some rather random set of points are color 1, some of the others are randomly assigned to color 2, and so on until every point has one of the colors. If there are n-many colors, they are named 1, 2, etc. out to n.

"Circles" are the usual circles on the sphere. A circle is determined by any set of three points, in the sense that fixing three points, there is precisely one circle that contains all of them. As a result, given any three points of different colors there is a circle that

[1] Lest this seem excessively noble on my part, let me add that what I actually do for a living is to work as an independent speculator in the financial markets, where my colleagues constitute what is probably the greediest profession in the world.

contains those three colors. We know from earlier work that there has to be a circle that has four of the colors on it. It is impossible to color the sphere is a way that never puts four colors on any circle, even though for any given circle three colors will typically be enough. Now a further question poses itself. What if there are six or seven, or m different colors. What can we say about the number of circles there must be having any given (smaller) number of colors on them? In this context, we have to recognize that the terms of the question have shifted in an important way. When we ask "How many circles…?" we are not really counting circles per se. We are counting *kinds* of circles. So for instance if there are six different colors, and we ask how many circles have four colors on them, we are really asking the following question:

> Represent an actual circle by the set of colors that occur on it, so a circle that has colors 1, 2, 3, and 4 on it is simply represented as the set {1,2,3,4}, and agree that when we say "different circles" we really mean "different sets of colors." Then how many different circles must there be if no circles (no actual, round circle) has more than four colors on it.

In the case of these particular numbers – six colors in total but no more than four of them on any given actual spherical circle – there must be at least three different kinds of four colored circles. There must be at least three different four-tuples of colors corresponding to the colors on actual circles. So for instance, there can be circles {1,2,3,4}, {1,2,5,6}, and {3,4,5,6}.How many actual circles there are with these three configurations of colors on them is impossible to say. All we know for sure is that there has to be at least one of each of them.

The reason why this is necessary is actually very easily explained. Starting with the six original colors, imagine modifying the coloring by simply merging colors 1 and 2. This leaves a five-colored sphere. One of the colors is "1+2" – the color that merges the sets of points of colors 1 and 2 – and the others are 3, 4, 5, and 6 as before. Now this coloring by five colors must leave one circle with four of the on it. The four colored circle could have the colors "1+2", 3, 4, and 5, to take an example, and then the circle with those colors has to have come from a circle that had either 1, 3, 4, and 5 OR 2, 3, 4, and 5. There is by assumption no circle with five colors on it, so this is an either/or matter. Suppose the four colored circle is {1,3,4,5}. Now imagine merging colors 1 and 3. That would leave a five colored sphere with colors “1+3”, 2, 4, 5, and 6. The circle we just identified, {1,3,4,5} would have only three colors on it, “1+3”, 4, and 5. Unless there is another four-tuple of colors on a circle somewhere, we would violate the known fact that in this coloring there has to be a four colored circle somewhere. So, the upshot of this mental calculation is that there must be at least three different four-tuples of colors, each occurring on three different circles. So by applying this logic of merging colors we can in theory compute the absolute minimal number of circles having any given number of colors. One of the complexities of this problem is that there is no single actual minimal collection of circles. Many different arrangements of colors and circles all reach the same minimal *number* of circles, which being very different actual circles. The obvious approach to the problem that consists of finding *the* collection of circles that achieves the minimum does not work. There is no single, minimizing set. There is only a unique minimum number, but many different ways to get it.

Now I recognize that not everyone who has the good fortune to peruse this essay will have found the forgoing reasoning as fascinating as I do, but for them I especially want to offer my assurance that this is about to get really interesting. What in this case is so interesting is not the calculations,[2] but the method of carrying them out. Specifically, it is a practical, human necessity to have some kind of tool with which to keep track of the calculations, and it is that tool that needs to be explained here. I dragged the reader through a part of the calculation in detail in order to build an appreciation of where the tool comes from. “Tools,” as I am using the term here, are logical constructs that reduce reasoning to feasible sequences to steps, and that illuminate generalizations that stand out as applying to the whole problem.

We have noted already that while this problem is related to circles on a sphere, in order to tackle it in practice we dispense with actual circles, while focusing on characters that we identify with colors, and on sets of these characters. From these “letters,” as it were, we build “words,” which are understood as representing color combinations on ac-

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[2] In mathematical terminology, reasoning toward the solution of a problem which, though perhaps very difficult, is known with certainty to have a solution is called a “calculation.” In the case of questions for which there is no assurance that any solution even exists are called “existence proofs.” “Pure” math focuses on that kind of problem. As a result, mathematics, which in practical terms is the science that provides the world with methods of calculation, has itself no intrinsic method of calculation. Every problem solved is a triumph of luck over improbability.

tual circles. The words are then surveyed for feasibility, necessity (because we want to keep only the minimal number of different ones), and any other pertinent criteria. The way I implemented this was by way of a matrix in which the columns correspond to colors and the rows correspond to circles. As an example, the following matrix represents a coloring of the sphere which uses six colors, and three circles on the sphere that each have four of the colors on them:

| | *1* | *2* | *3* | *4* | *5* | *6* |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 2 | 1 | 1 | 0 | 0 | 1 | 1 |
| 3 | 0 | 0 | 1 | 1 | 1 | 1 |

The numbers down the left column count circles while the numbers across the top count colors. The entries are always zeroes or ones, where "1" designates the presence of a color and "0" its absence. The first row refers to a circle that contains colors 2, 3, 4, and 5: {2,3,4,5}. The second row is a circle {1,2,5,6} and the third row is the circle {3,4,5,6}. The reasoning illustrated previously implies that this collection of circles is a solution to the initial question, in the sense that it is a feasible set of circles on the sphere, and it is minimal because no two of the three circles would be a solution. If for instance we dispensed with the third circle, both of the remaining circles would have the colors 1 and 2 on them. But in that case, merging 1 and 2 would leave no four-colored circle at all.

So? Fascinating you say. But we learn two general properties of this kind of matrix. First is how we merge colors. To merge colors 3 and 5, for instance, we simply add across each row. This leaves a new matrix that has one less column – we shift columns to the left – in which some of the rows are no longer valid solutions. A row ceases in this way to be a solution when it ceases to have exactly four ones in it. The result of merging colors 3 and 5 ill shown below:

| | *1* | *2* | *3* | *4* | *5* |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 0 |
| 2 | 1 | 1 | 1 | 0 | 1 |
| 3 | 0 | 0 | 2 | 1 | 1 |

The first two rows are still valid solutions (colorings), but the third row is not. The third row of the original table is still part of a valid arrangement of colors on circles, and the whole table is still a valid solution, because all that is necessary as that *at least one* row be valid. The other generalization is that for a given matrix to correspond to a valid arrangement of colors and circles, each row has the property that there is some sequence of merges – if there were more columns the sequences would be clearer – under which only that row remains a valid solution (i.e. for which that row and only that row still have exactly four ones and the rest zeroes). If a given sequence of merges left two valid circles, we could simply dispense with one of them.

| | *1* | *2* | *3* | *4* |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 2 | 1 | 0 |
| 3 | 0 | 1 | 2 | 1 |

The acid test of course is when we merge colors to the point where only four remain. The matrix shows, for instance, the effect of first merging colors 3 and 5, as before, and then merging colors 2 and 5. Note that "5" in this case is actually the merger of the original colors "3" and "5", but that is perfectly okay. Only the first row is valid. There are many possible mergers, for six colors there are fifteen possible merges down from six to five columns. Some of these sequences leave no valid rows, but that is not catastrophic. What is necessary is that every row contains a coloring that remains valid after some sequence of merges and that no other row survives that particular sequence. The first of these propositions implies that, since all possible merges are feasible and, in some sense, "allowed," the row in question is essential – it is necessitated by a given sequence of merges, and no other row could replace it. The latter condition insures that the whole arrangement is minimal.

There are other generalizations that follow logically from these, but these are enough for our purposes. The generalizations are necessary because we wish to draw general conclusions, but specific matrices do not generalize. Only the common rules do. That tells us something about what we need to do to tackle this problem. We need to have a tool, for which in this case the matrix and all the abstractions by which we derived it from an actual coloring of the sphere serve. And we need to be able to *think about* the tool, as a mental construct in itself. My small example is too small to illustrate this adequately, but I will simply decree what it means here. We now know a matrix that "works" to illustrate the solution to the problem of six colors and no more than four on any circle, but we want full generality give me a matrix that corresponds to a valid arrangement for 2,800 colors and no more than 77 colors on any circle. That matrix is gigantic, so no one is actually proposing to write it out. We need another kind of generalization, one which gives rules that tell us how we would get from any matrix solution for given k (colors on a circle) and m (total number of colors), to k+1, and separately to m+1. And the rule has to enable us to verify that the expanded matrix *necessarily* complies with the generalizations of feasibility and minimality.[3]

So now we need to study matrices in order to extract information about their necessary, general properties. We do that by constructing matrices, starting with small k's and m's, and searching for those generalizations. But when I say "constructing," I mean actually constructing. Obviously pencil and paper, or better yet computer spreadsheets, are very useful. But they are not enough. The program of study consists of constructing in out neurons replicas of these examples. The paper cannot point to generalizations. We have to be actually *thinking about* the matrices, and the matrices are real things (in a rather abstract, mathematical use of the term "real"). Only other real things – neurons – can replicate real matrices. What is fascinating here is that we do not merely remember the matrices as examples. We *study* them. We touch them and weigh them and turn them over on their back sides. We prod and probe and stretch them until they tell us what we want to know. The evidence of this is that when we have finished, we remember them because they are now neural constructs in our cortex. A part of our body actually is the family of matrices that we studied in search of the generalizations we were looking for. To the degree that you will agree that the matrices are real, we have reconfigured a little of the available space amongst our neurons to build a complete replica.

This is an astonishing capacity that we have. But in another sense it is quite unastonishing. It is astonishing when we contemplate the results that we achieve by it, that

[3] Recall that feasibility means that every sequence of merges leaves at least one valid row and minimality means that for every row there is precisely one sequence of merges that leave it as the only valid row.

could not be achieved in any other way. But it is quite un-astonishing when we stop to think that chimpanzees, at the very least, have all the neural capacity that we have. There is no biological reason why they don't solve these problems just as fast as we do. In fact, since their lives in nature are shall we say rather un-demanding, from the point of view of let's say how many hours they waste each day commuting, we could ponder why it is they don't ever seem to solve even the easy ones. What are we missing? Well, more to the point, what are *they* missing? They seem to lack the concept of *problem*. Not to say that they don't confront what we call problems. "Problems" are a reality. Every living thing confronts and sometimes solves them. As we agree that the chimpanzees amazing neural capacity really helps them greatly in surmounting the problems that arise. So some measure to neural competence is essential, and more is better. They are just like us in those respects.

But they lack something. And it is something *Big*. They lack the concept of *concepts*. The difference between them and us could hardly by more stark, because it is not identifiably one of ability. The chimps have all the neural tools to think about anything. If it took tools, they would be as smart as we are. There is nothing that we are able to think about that they are not also able to think about. Their neurons are as plastic as our are. They can be *taught*, in the sense of being configured to replicate the same sort of situations in reality that they or we need to be able to replicate and deal with. And they have memory, as we do. Whatever they learn, whatever they embed in their neurons, remains and remains accessible to them. They have all the tools of thought. They just don't *Actually* think.[4] They aren't aware that they are missing anything, and again I emphasize that they do use many of the fruits of thought and learning, which is to say they are trainable – even pigeons are trainable – and trainable by their life experiences. They learn to do things that are rewarded by life and to avoid things that are punished. They just don't have the concept that learning is something one would *want* to do. In fact, it would not be unfair to say that there is nothing that they *want* to do.

When we say that we want to do this or that we mean that we are imagining a plan to do it. Even if we are not far enough along to have the plan, we at least recognize what

[4] We of course actually think about much harder problems than they do, but it is not clear that they are constitutionally unable to think about them. It is quite possible that even rats can be conditioned to solve differential equations.

it would be like to have one, and we want to get there if at all possible. There is no meaning of wants except in conjunction with plans, if even abstract ones. Anyone who is unable to contemplate what a plan is cannot be said to want anything. The plan can be part of the *want*; it can be the first thing we want and don't have idea of how to get. But we have to have the concept of using our tools to achieve ends. Since the chimps don't have any conscious ends, they have no concept of intentionally using their impressive tools to achieve them. Human beings then have a unique skill, if that is the correct term for it, which is to have concepts. And the difference is not a quantitative difference. We don't have *more* concepts than chimps do. Our difference is that they have none, and on our side we *always* have concepts. We cannot think about thinking without thinking about what concepts we would need to have. For us, thinking simply *means* having concepts. The contrast is so stark that, ironically, it enables us to understand the difference very easily. We can imagine what it would be like not to have concepts. It would be like death, or at the least it would be like living in a trance. We should think of chimpanzees as creatures with big brain who are strung out on drugs all the time. We can understand that. We just can't understand *actually being* that. The very moment we start to think, we start to think about concepts. We can't help it, but then our empathy for chimps flies right out the window.

The point, in the end, is that this is the meaning of mind and will. Mind does not so much determine what we are able to learn or to think about. Mind defines what it is that we *actually do* think about. Mind and will are the thinking chimp. Why we have concepts and chimps do not is a deep mystery because there is nothing palpable that we have that they do not have, but yet the gap between them and us is deeper than the gap between them and sponges. This bears further thought.

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